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### Abstract

This thesis is a blend of neutron transport theory and numerical analysis. We start with the study of the problem of the Mika/Case eigenexpansion used in the solution process of the homogeneous one-speed Boltzmann neutron transport equation with anisotropic scattering for plane symmetry. The anisotropic scattering is expressed as a finite Legendre series in which the coefficients are the 'scattering coefficients'. This eigenexpansion consists of a discrete spectrum of eigenvalues with its corresponding eigenfunctions and the continuous spectrum  $[-1,+1]$  with its corresponding eigendistributions. In the general case where the anisotropic scattering can be of any (finite) order, multiple discrete eigenvalues exist and these have to be located to have the complete spectrum. We have devised a stable and robust method that locates all these discrete eigenvalues. The method is a two-step process: first the number of discrete eigenvalues is calculated and this is followed by the calculation of the discrete eigenvalues themselves, now being able to count them down and make sure none are forgotten.

During our numerical experiments, we came across what we called near-singular eigenvalues: discrete eigenvalues that are located extremely close to the continuum and hence lead to near-singular behaviour in the eigenfunction. Our solution method has been adapted and allows for the automatic detection of such a near-singular eigenvalue.

For the elements of the continuous spectrum  $[-1,+1]$ , there is no non-zero function satisfying the associated eigenequation but there is a non-zero distribution that does satisfy it. It is not feasible to compute a distribution as such but one can evaluate integrals in which this distribution appears. The continuum part of the eigenexpansion can hence only be characterised by its (angular) moments. Accurate and fast numerical quadrature is needed to evaluate these integrals. Several quadrature methods have been evaluated on a representative test function.

The eigenexpansion was proved to be orthogonal and complete and hence can be used to represent the infinite medium Green's function. The latter is the building block of the Boundary Sources Method, an integral solution method for the neutron transport equation. Using angular and angular/spatial moments of the Green's function, it is possible to solve with high accuracy slab

problems. We have written a one-dimensional slab code implementing this Boundary Sources Method allowing for media with arbitrary order anisotropic scattering. Our results are very good and the code can be considered as a benchmark code for others.

As a final application, we have used our code to study the discrete spectrum of a well-known scattering kernel in radiative transfer, the Henyey-Greenstein kernel. This kernel has one free parameter which is used to fit the kernel to experimental data. Since the kernel is a continuous function, a finite Legendre approximation needs to be adopted. Depending on the free parameter, the approximation order and the number of secondaries per collision, the number of discrete eigenvalues ranges from two to thirty and even more. Bounds for the minimum approximation order are derived for different requirements on the approximation: non-negativity, an absolute and relative error tolerance.